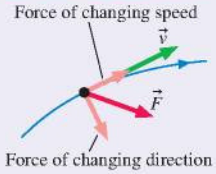
8 Dynamics II: Motion in a Plane

Why doesn't the roller coaster fall off the track at the top of the loop?

IN THIS CHAPTER, you will learn to solve problems about motion in two dimensions.

Are Newton's laws different in two dimensions?

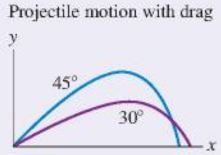
No. Newton's laws are vector equations, and they work equally well in two and three dimensions. For motion in a plane, we'll focus on how a force *tangent* to a particle's trajectory changes its speed, while a force *perpendicular* to the trajectory changes the particle's direction.



« LOOKING BACK Chapter 4 Kinematics of projectile and circular motion

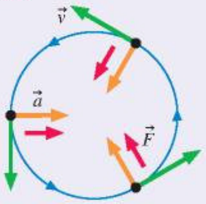
How do we analyze projectile-like motion?

For linear motion, one component of the acceleration was always zero. Motion in a plane generally has acceleration along two axes. If the accelerations are independent, we can use *x*- and *y*-coordinates and we will find motions analogous to the projectile motion we studied in Chapter 4.



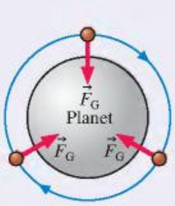
How do we analyze circular motion?

Circular motion must have a force component toward the center of the circle to create the centripetal acceleration. In this case the acceleration components are radial and, perhaps, tangential. We'll use a different coordinate system, *rtz* coordinates, to study the dynamics of circular motion



Does this analysis apply to orbits?

Yes, it does. The circular orbit of a satellite or planet is motion in which the force of gravity is creating the inward centripetal acceleration. You'll see that an orbiting projectile is in free fall.



« LOOKING BACK Section 6.3 Gravity and weight

Why doesn't the water fall out of the bucket?

How can you swing a bucket of water over your head without the water falling out? Why doesn't a car going around a loop-the-loop fall off at the top? Circular motion is not always intuitive, but you'll strengthen your ability to use Newtonian reasoning by thinking about some of these problems.

Why is planar motion important?

By starting with linear motion, we were able to develop the ideas and tools of Newtonian mechanics with minimal distractions. But planes and rockets move in a plane. Satellites and electrons orbit in a plane. The points on a rotating hard drive move in a plane. In fact, much of this chapter is a prelude to Chapter 12, where we will study rotational motion. This chapter gives you the tools you need to analyze more complex—and more realistic—forms of motion.

8.1 Dynamics in Two Dimensions

Newton's second law, $\vec{a} = \vec{F}_{net}/m$, determines an object's acceleration; it makes no distinction between linear motion and two-dimensional motion in a plane. We began with motion along a line, in order to focus on the essential physics, but now we turn our attention to the motion of projectiles, satellites, and other objects that move in two dimensions. We'll continue to follow « Problem-Solving Strategy 6.1, which is well worth a review, but we'll find that we need to think carefully about the appropriate coordinate system for each problem.

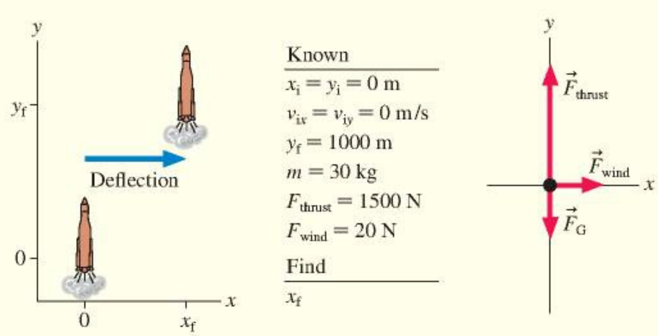
EXAMPLE 8.1 | Rocketing in the wind

A small rocket for gathering weather data has a mass of 30 kg and generates 1500 N of thrust. On a windy day, the wind exerts a 20 N horizontal force on the rocket. If the rocket is launched straight up, what is the shape of its trajectory, and by how much has it been deflected sideways when it reaches a height of 1.0 km? Because the rocket goes much higher than this, assume there's no significant mass loss during the first 1.0 km of flight.

MODEL Model the rocket as a particle. We need to find the *function* $y(x)$ describing the curve the rocket follows. Because rockets have aerodynamic shapes, we'll assume no vertical air resistance.

VISUALIZE FIGURE 8.1 shows a pictorial representation. We've chosen a coordinate system with a vertical y-axis. Three forces act on the rocket: two vertical and one horizontal.

FIGURE 8.1 Pictorial representation of the rocket launch.



SOLVE In this problem, the vertical and horizontal forces are independent of each other. Newton's second law is

$$a_x = \frac{(F_{net})_x}{m} = \frac{F_{wind}}{m}$$

$$a_y = \frac{(F_{net})_y}{m} = \frac{F_{thrust} - mg}{m}$$

The primary difference from the linear-motion problems you've been solving is that the rocket accelerates along both axes. However, both accelerations are constant, so we can use kinematics to find

$$x = \tfrac{1}{2}a_x(\Delta t)^2 = \frac{F_{wind}}{2m}(\Delta t)^2$$

$$y = \tfrac{1}{2}a_y(\Delta t)^2 = \frac{F_{thrust} - mg}{2m}(\Delta t)^2$$

where we used the fact that all initial positions and velocities are zero. From the x-equation, $(\Delta t)^2 = 2mx/F_{wind}$. Substituting this into the y-equation, we find

$$y(x) = \left(\frac{F_{thrust} - mg}{F_{wind}}\right)x$$

This is the equation of the rocket's trajectory. It is a linear equation. Somewhat surprisingly, given that the rocket has both vertical and horizontal accelerations, its trajectory is a *straight line*. We can rearrange this result to find the deflection at height y:

$$x = \left(\frac{F_{wind}}{F_{thrust} - mg}\right)y$$

From the data provided, we can calculate a deflection of 17 m at a height of 1000 m.

ASSESS The solution depended on the fact that the time parameter Δt is the *same* for both components of the motion.

Projectile Motion

We found in Chapter 6 that the gravitational force on an object near the surface of a planet is $\vec{F}_G = (mg, \text{down})$. For a coordinate system with a vertical y-axis,

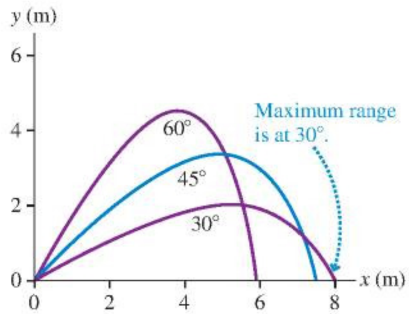
$$\vec{F}_G = -mg\hat{j} \tag{8.1}$$

Consequently, from Newton's second law, the acceleration is

$$a_x = \frac{(F_G)_x}{m} = 0$$
$$a_y = \frac{(F_G)_y}{m} = -g \tag{8.2}$$

Equations 8.2 justify the analysis of projectile motion in « Section 4.2—a downward acceleration $a_y = -g$ with no horizontal acceleration—where we found that a drag-free

FIGURE 8.2 A projectile is affected by drag. These are trajectories of a plastic ball launched at different angles.



projectile follows a parabolic trajectory. The fact that the two components of acceleration are independent of each other allows us to solve for the vertical and horizontal motions.

However, the situation is quite different for a low-mass projectile, where the effects of drag are too large to ignore. We'll leave it as a homework problem for you to show that the acceleration of a projectile subject to drag is

$$\begin{aligned} a_x &= -\frac{\rho C A}{2m} v_x \sqrt{v_x^2 + v_y^2} \\ a_y &= -g - \frac{\rho C A}{2m} v_y \sqrt{v_x^2 + v_y^2} \end{aligned} \tag{8.3}$$

Here the components of acceleration are neither constant nor independent of each other because a_x depends on v_y and vice versa. It turns out that these two equations cannot be solved exactly for the trajectory, but they can be solved numerically. FIGURE 8.2 shows the numerical solution for the motion of a 5 g plastic ball that's been hit with an initial speed of 25 m/s. It doesn't travel very far (the maximum distance without drag would be more than 60 m), and the maximum range is no longer reached for a launch angle of 45°. Notice that the trajectories are not at all parabolic.

STOP TO THINK 8.1 This force will cause the particle to

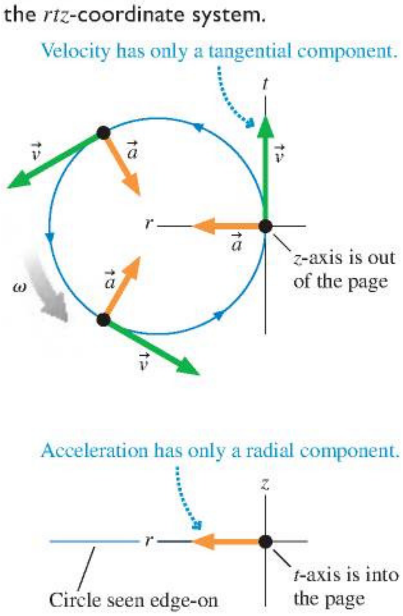
a. Speed up and curve upward.
b. Speed up and curve downward.
c. Slow down and curve upward.
d. Slow down and curve downward.
e. Move to the right and down.
f. Reverse direction.

8.2 Uniform Circular Motion

The kinematics of uniform circular motion were introduced in « Sections 4.4–4.5, and a review is *highly* recommended. Now we're ready to study *dynamics*—how forces *cause* circular motion. FIGURE 8.3 reminds you that the particle's velocity is tangent to the circle, and its acceleration—a *centripetal acceleration*—points toward the center. If the particle has angular velocity ω and speed $v = \omega r$, its centripetal acceleration is

$$\vec{a} = \left(\frac{v^2}{r}, \text{ toward center of circle}\right) = (\omega^2 r, \text{ toward center of circle}) \tag{8.4}$$

FIGURE 8.3 Uniform circular motion and the *rtz*-coordinate system.



An *xy*-coordinate system is not a good choice to analyze circular motion because the *x*- and *y*-components of the acceleration are not constant. Instead, as Figure 8.3 shows, we'll use a coordinate system whose axes are defined as follows:

- The origin is at the point where the particle is located.
- The *r*-axis (radial axis) points *from* the particle *toward* the center of the circle.
- The *t*-axis (tangential axis) is tangent to the circle, pointing in the counterclockwise direction.
- The *z*-axis is perpendicular to the plane of motion.

These three mutually perpendicular axes form the **rtz-coordinate system.**

You can see that the *rtz*-components of $\vec{v}$ and $\vec{a}$ are

$$\begin{aligned} v_r &= 0 \qquad & a_r &= \frac{v^2}{r} = \omega^2 r \\ v_t &= \omega r \qquad & a_t &= 0 \\ v_z &= 0 \qquad & a_z &= 0 \end{aligned} \tag{8.5}$$

where the angular velocity $\omega = d\theta/dt$ must be in rad/s. For uniform circular motion, **the velocity vector has only a tangential component and the acceleration vector has only a radial component.** Now you can begin to see the advantages of the *rtz*-coordinate system.

NOTE Recall that ω and v_t are positive for a counterclockwise (ccw) rotation, negative for a clockwise (cw) rotation. The particle's speed is $v = |v_t|$.

Dynamics of Uniform Circular Motion

A particle in uniform circular motion is clearly not traveling at constant velocity in a straight line. Consequently, according to Newton's first law, the particle *must* have a net force acting on it. We've already determined the acceleration of a particle in uniform circular motion—the centripetal acceleration of Equation 8.4. Newton's second law tells us exactly how much net force is needed to cause this acceleration:

$$\vec{F}_{\text{net}} = m\vec{a} = \left(\frac{mv^2}{r}\text{, toward center of circle}\right) \tag{8.6}$$

In other words, a particle of mass m moving at constant speed v around a circle of radius r *must* have a net force of magnitude mv^2/r pointing toward the center of the circle. Without such a force, the particle would move off in a straight line tangent to the circle.

FIGURE 8.4 shows the net force $\vec{F}_{\text{net}}$ acting on a particle as it undergoes uniform circular motion. You can see that **$\vec{F}_{\text{net}}$, like $\vec{a}$, points along the radial axis of the *rtz*-coordinate system, toward the center of the circle.** The tangential and perpendicular components of $\vec{F}_{\text{net}}$ are zero.

NOTE The force described by Equation 8.6 is not a *new* force. The force itself must have an identifiable agent and will be one of our familiar forces, such as tension, friction, or the normal force. Equation 8.6 simply tells us how the force needs to act—how strongly and in which direction—to cause the particle to move with speed v in a circle of radius r.

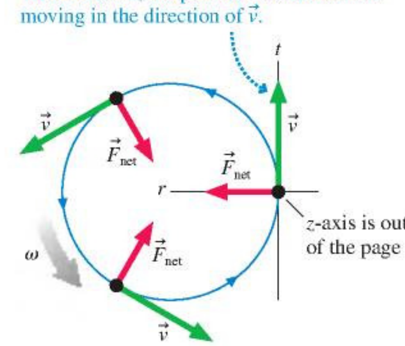
The usefulness of the *rtz*-coordinate system becomes apparent when we write Newton's second law, Equation 8.6, in terms of the r-, t-, and z-components:

$$\begin{aligned}(F_{\text{net}})_r &= \sum F_r = ma_r = \frac{mv^2}{r} = m\omega^2 r\\ (F_{\text{net}})_t &= \sum F_t = ma_t = 0\\ (F_{\text{net}})_z &= \sum F_z = ma_z = 0\end{aligned} \tag{8.7}$$

For uniform circular motion, the sum of the forces along the t-axis and along the z-axis must equal zero, and the sum of the forces along the r-axis *must* equal ma_r, where a_r is the centripetal acceleration.

On banked curves, the normal force of the road assists in providing the centripetal acceleration of the turn.

FIGURE 8.4 The net force points in the radial direction, toward the center of the circle.



EXAMPLE 8.2 Spinning in a circle

An energetic father places his 20 kg child on a 5.0 kg cart to which a 2.0-m-long rope is attached. He then holds the end of the rope and spins the cart and child around in a circle, keeping the rope parallel to the ground. If the tension in the rope is 100 N, how many revolutions per minute (rpm) does the cart make? Rolling friction between the cart's wheels and the ground is negligible.

MODEL Model the child in the cart as a particle in uniform circular motion.

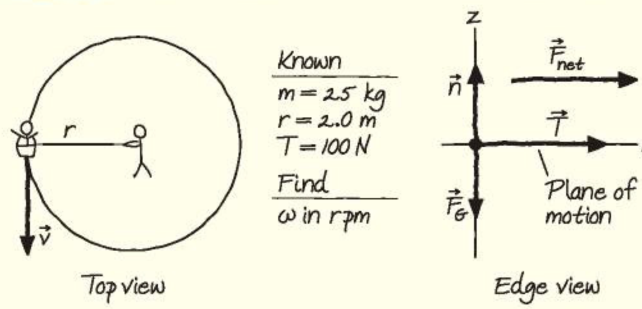
VISUALIZE **FIGURE 8.5** on the next page shows the pictorial representation. A circular-motion problem usually does not have starting and ending points like a projectile problem, so numerical subscripts such as x_1 or y_2 are usually not needed. Here we need to define the cart's speed v and the radius r of the circle. Further, a motion diagram is not needed for uniform circular motion because we already know the acceleration $\vec{a}$ points to the center of the circle.

The essential part of the pictorial representation is the free-body diagram. **For uniform circular motion we'll draw the free-body diagram in the *rz*-plane, looking at the edge of the circle, because this is the plane of the forces.** The contact forces acting on the cart are the normal force of the ground and the tension force of the rope. The normal force is perpendicular to the plane of the motion and thus in the z-direction. The direction of $\vec{T}$ is determined by the statement that the rope is parallel to the ground. In addition, there is the long-range gravitational force $\vec{F}_G$.

SOLVE We defined the r-axis to point toward the center of the circle, so $\vec{T}$ points in the positive r-direction and has r-component $T_r = T$.

Continued

FIGURE 8.5 Pictorial representation of a cart spinning in a circle.



Newton's second law, using the *rtz*-components of Equations 8.7, is

$$\sum F_r = T = \frac{mv^2}{r}$$
$$\sum F_z = n - mg = 0$$

We've taken the *r*- and *z*-components of the forces directly from the free-body diagram, as you learned to do in Chapter 6. Then we've *explicitly* equated the sums to $a_r = v^2/r$ and $a_z = 0$. This is the basic strategy for all uniform circular-motion problems. From the *z*-equation we can find that $n = mg$. This would be useful if we needed to determine a friction force, but it's not needed in this problem. From the *r*-equation, the speed of the cart is

$$v = \sqrt{\frac{rT}{m}} = \sqrt{\frac{(2.0\text{ m})(100\text{ N})}{25\text{ kg}}} = 2.83\text{ m/s}$$

The cart's angular velocity is then found from Equations 8.5:

$$\omega = \frac{v_t}{r} = \frac{v}{r} = \frac{2.83\text{ m/s}}{2.0\text{ m}} = 1.41\text{ rad/s}$$

This is another case where we inserted the radian unit because ω is specifically an *angular* velocity. Finally, we need to convert ω to rpm:

$$\omega = \frac{1.41\text{ rad}}{1\text{ s}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}} = 14\text{ rpm}$$

ASSESS 14 rpm corresponds to a period $T \approx 4$ s. This result is reasonable.

The Central-Force Model

A force that is always directed toward the same point is called a **central force.** The tension in the rope of the last example is a central force, as is the gravitational force acting on an orbiting satellite. An object acted on by an attractive central force can undergo uniform circular motion around the central point. More complicated trajectories can occur in some situations—such as satellites following elliptical orbits—but for now we'll focus on circular motion, or motion with constant *r*. This **central-force model** is another important model of motion.

MODEL 8.1

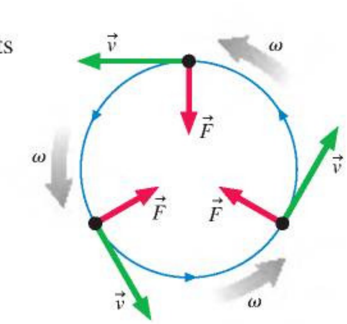
Central force with constant r

For objects on which a constant net force points toward a central point.

- Model the object as a particle.
- The force causes a centripetal acceleration.
  - The motion is uniform circular motion.
- Mathematically:
  - Newton's second law is

$$\vec{F}_{\text{net}} = \left(\frac{mv^2}{r} \text{ or } m\omega^2 r\text{, toward center}\right)$$

  - Use the kinematics of uniform circular motion.
- Limitations: Model fails if the force has a tangential component or if *r* changes.



The object undergoes uniform circular motion.

Exercise 10

Let's look at more examples of the central-force model in action.

EXAMPLE 8.3 Turning the corner I

What is the maximum speed with which a 1500 kg car can make a left turn around a curve of radius 50 m on a level (unbanked) road without sliding?

MODEL The car doesn't complete a full circle, but it is in uniform circular motion for a quarter of a circle while turning. We can model the car as a particle subject to a central force. Assume that rolling friction is negligible.

VISUALIZE FIGURE 8.6 shows the pictorial representation. The issue we must address is *how* a car turns a corner. What force or forces cause the direction of the velocity vector to change? Imagine driving on a completely frictionless road, such as a very icy road. You would not be able to turn a corner. Turning the steering wheel would be of no use; the car would slide straight ahead, in accordance with both Newton's first law and the experience of anyone who has ever driven on ice! So it must be *friction* that somehow allows the car to turn.

Figure 8.6 shows the top view of a tire as it turns a corner. If the road surface were frictionless, the tire would slide straight ahead. The force that prevents an object from sliding across a surface is *static friction.* Static friction $\vec{f}_s$ pushes *sideways* on the tire, toward the center of the circle. How do we know the direction is sideways? If $\vec{f}_s$ had a component either parallel to $\vec{v}$ or opposite to $\vec{v}$, it would cause the car to speed up or slow down. Because the car changes direction but not speed, static friction must be perpendicular to $\vec{v}$. $\vec{f}_s$ causes the centripetal acceleration of circular motion around the curve, and thus the free-body diagram, drawn from behind the car, shows the static friction force pointing toward the center of the circle.

SOLVE The maximum turning speed is reached when the static friction force reaches its maximum $f_{s\,max} = \mu_s n$. If the car enters the curve at a speed higher than the maximum, static friction will not be large enough to provide the necessary centripetal acceleration and the car will slide.

The static friction force points in the positive r-direction, so its radial component is simply the magnitude of the vector: $(f_s)_r = f_s$. Newton's second law in the rtz-coordinate system is

$$\sum F_r = f_s = \frac{mv^2}{r}$$

$$\sum F_z = n - mg = 0$$

The only difference from Example 8.2 is that the tension force toward the center has been replaced by a static friction force toward the center. From the radial equation, the speed is

$$v = \sqrt{\frac{rf_s}{m}}$$

The speed will be a maximum when f_s reaches its maximum value:

$$f_s = f_{s\,max} = \mu_s n = \mu_s mg$$

where we used $n = mg$ from the z-equation. At that point,

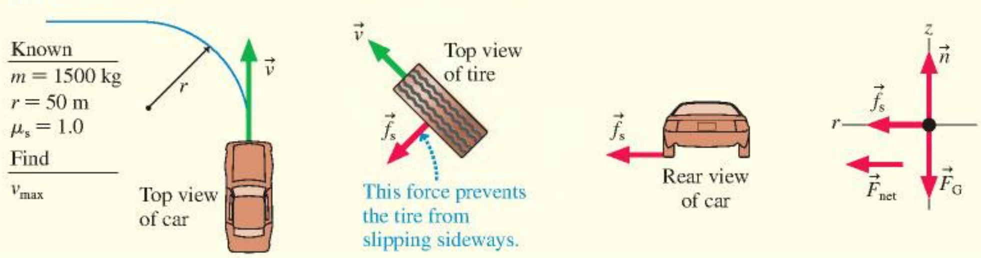
$$v_{max} = \sqrt{\frac{rf_{s\,max}}{m}} = \sqrt{\mu_s rg}$$

$$= \sqrt{(1.0)(50\text{ m})(9.80\text{ m/s}^2)} = 22\text{ m/s}$$

where the coefficient of static friction was taken from Table 6.1.

ASSESS 22 m/s ≈ 45 mph, a reasonable answer for how fast a car can take an unbanked curve. Notice that the car's mass canceled out and that the final equation for v_{max} is quite simple. This is another example of why it pays to work algebraically until the very end.

FIGURE 8.6 Pictorial representation of a car turning a corner.



Because μ_s depends on road conditions, the maximum safe speed through turns can vary dramatically. Wet roads, in particular, lower the value of μ_s and thus lower the speed of turns. Icy conditions are even worse. The corner you turn every day at 45 mph will require a speed of no more than 15 mph if the coefficient of static friction drops to 0.1.

EXAMPLE 8.4 Turning the corner II

A highway curve of radius 70 m is banked at a 15° angle. At what speed v_0 can a car take this curve without assistance from friction?

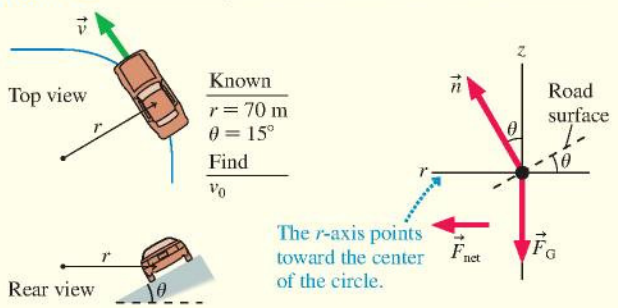
MODEL Model the car as a particle subject to a central force.

VISUALIZE Having just discussed the role of friction in turning corners, it is perhaps surprising to suggest that the same turn can also be accomplished without friction. Example 8.3 considered a level roadway, but real highway curves are *banked* by being tilted

Continued

up at the outside edge of the curve. The angle is modest on ordinary highways, but it can be quite large on high-speed racetracks. The purpose of banking becomes clear if you look at the free-body diagram in FIGURE 8.7. The normal force $\vec{n}$ is perpendicular to the road, so tilting the road causes $\vec{n}$ to have a component toward the center of the circle. **The radial component n_r is the central force that causes the centripetal acceleration needed to turn the car.** Notice that we are *not* using a tilted coordinate system, although this looks rather like an inclined-plane problem. The center of the circle is in the same horizontal plane as the car, and for circular-motion problems we need the r-axis to pass through the center. Tilted axes are for *linear* motion along an incline.

FIGURE 8.7 Pictorial representation of a car on a banked curve.



SOLVE Without friction, $n_r = n \sin\theta$ is the only component of force in the radial direction. It is this inward component of the normal force on the car that causes it to turn the corner. Newton's second law is

$$\sum F_r = n \sin\theta = \frac{mv_0^2}{r}$$
$$\sum F_z = n \cos\theta - mg = 0$$

where θ is the angle at which the road is banked and we've assumed that the car is traveling at the correct speed v_0. From the z-equation,

$$n = \frac{mg}{\cos\theta}$$

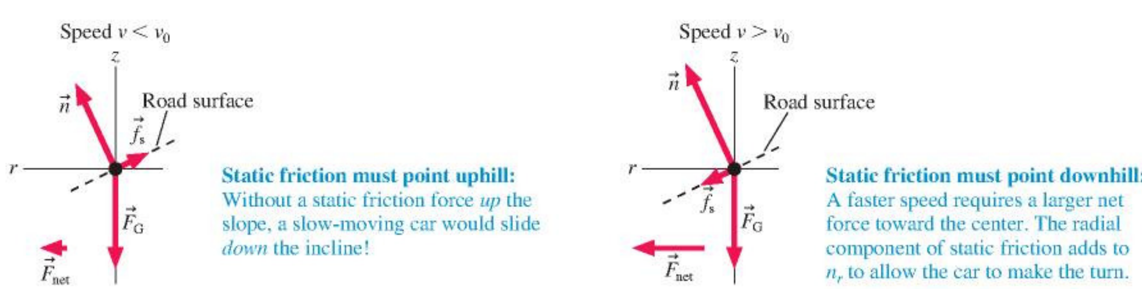
Substituting this into the r-equation and solving for v_0 give

$$\frac{mg}{\cos\theta}\sin\theta = mg\tan\theta = \frac{mv_0^2}{r}$$
$$v_0 = \sqrt{rg\tan\theta} = 14 \text{ m/s}$$

ASSESS This is $\approx$ 28 mph, a reasonable speed. Only at this very specific speed can the turn be negotiated without reliance on friction forces.

It's interesting to explore what happens at other speeds on a banked curve. FIGURE 8.8 shows that the car will need to rely on both the banking *and* friction if it takes the curve at a speed faster or slower than v_0.

FIGURE 8.8 Free-body diagrams for a car going around a banked curve at speeds slower and faster than the friction-free speed v_0.



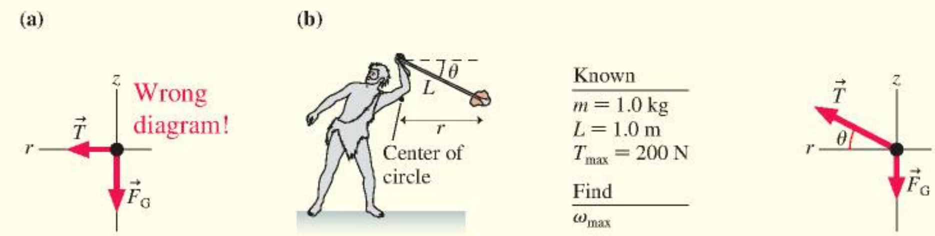
EXAMPLE 8.5 A rock in a sling

A Stone Age hunter places a 1.0 kg rock in a sling and swings it in a horizontal circle around his head on a 1.0-m-long vine. If the vine breaks at a tension of 200 N, what is the maximum angular speed, in rpm, with which he can swing the rock?

MODEL Model the rock as a particle in uniform circular motion.

VISUALIZE This problem appears, at first, to be essentially the same as Example 8.2, where the father spun his child around on a rope. However, the lack of a normal force from a supporting surface makes a *big* difference. In this case, the *only* contact force on the rock is the tension in the vine. Because the rock moves in a horizontal circle, you may be tempted to draw a free-body diagram like FIGURE 8.9a, where $\vec{T}$ is directed along the r-axis. You will quickly run into trouble, however, because this diagram has a net force in the z-direction and it is impossible to satisfy $\sum F_z = 0$. The

FIGURE 8.9 Pictorial representation of a rock in a sling.



gravitational force $\vec{F}_G$ certainly points vertically downward, so the difficulty must be with $\vec{T}$.

As an experiment, tie a small weight to a string, swing it over your head, and check the *angle* of the string. You will quickly discover that the string is *not* horizontal but, instead, is angled downward. The sketch of **FIGURE 8.9b** labels the angle θ. Notice that the rock moves in a *horizontal* circle, so the center of the circle is *not* at his hand. The r-axis points to the center of the circle, but the tension force is directed along the vine. Thus the correct free-body diagram is the one in Figure 8.9b.

SOLVE The free-body diagram shows that the downward gravitational force is balanced by an upward component of the tension, leaving the radial component of the tension to cause the centripetal acceleration. Newton's second law is

$$\sum F_r = T\cos\theta = \frac{mv^2}{r}$$

$$\sum F_z = T\sin\theta - mg = 0$$

where θ is the angle of the vine below horizontal. From the z-equation we find

$$\sin\theta = \frac{mg}{T}$$

$$\theta_{max} = \sin^{-1}\left(\frac{(1.0\text{ kg})(9.8\text{ m/s}^2)}{200\text{ N}}\right) = 2.81°$$

where we've evaluated the angle at the maximum tension of 200 N. The vine's angle of inclination is small but not zero.

Turning now to the r-equation, we find the rock's speed is

$$v_{max} = \sqrt{\frac{rT\cos\theta_{max}}{m}}$$

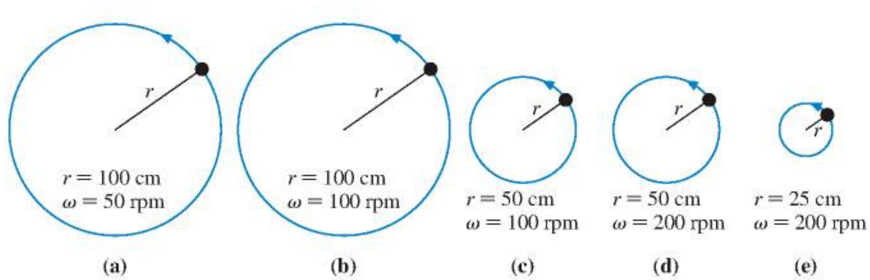
Careful! The radius r of the circle is *not* the length L of the vine. You can see in Figure 8.9b that $r = L\cos\theta$. Thus

$$v_{max} = \sqrt{\frac{LT\cos^2\theta_{max}}{m}} = \sqrt{\frac{(1.0\text{ m})(200\text{ N})(\cos 2.81°)^2}{1.0\text{ kg}}} = 14.1\text{ m/s}$$

We can now find the maximum angular speed, the value of ω that brings the tension to the breaking point:

$$\omega_{max} = \frac{v_{max}}{r} = \frac{v_{max}}{L\cos\theta_{max}} = \frac{14.1\text{ rad}}{1\text{ s}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}} = 135\text{ rpm}$$

STOP TO THINK 8.2 A block on a string spins in a horizontal circle on a frictionless table. Rank in order, from largest to smallest, the tensions T_a to T_e acting on blocks a to e.



8.3 Circular Orbits

Satellites orbit the earth, the earth orbits the sun, and our entire solar system orbits the center of the Milky Way galaxy. Not all orbits are circular, but in this section we'll limit our analysis to circular orbits.

How does a satellite orbit the earth? What forces act on it? To answer these important questions, let's return, for a moment, to projectile motion. Projectile motion occurs when the only force on an object is gravity. Our analysis of projectiles assumed that the earth is flat and that the acceleration due to gravity is everywhere straight down. This is an acceptable approximation for projectiles of limited range, such as baseballs or cannon balls, but there comes a point where we can no longer ignore the curvature of the earth.

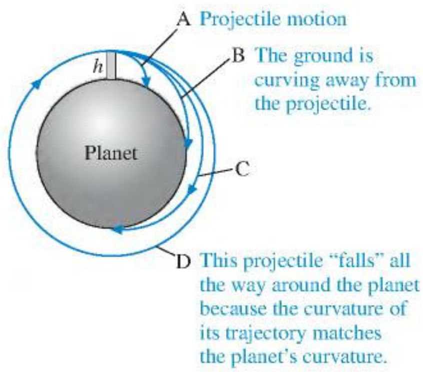


FIGURE 8.10 Projectiles being launched at increasing speeds from height h on a smooth, airless planet.

FIGURE 8.10 shows a perfectly smooth, spherical, airless planet with one tower of height h. A projectile is launched from this tower parallel to the ground ($\theta = 0°$) with speed v_0. If v_0 is very small, as in trajectory A, the "flat-earth approximation" is valid and the problem is identical to Example 8.4 in which a car drove off a cliff. The projectile simply falls to the ground along a parabolic trajectory.

As the initial speed v_0 is increased, the projectile begins to notice that the ground is curving out from beneath it. It is falling the entire time, always getting closer to the ground, but the distance that the projectile travels before finally reaching the ground—that is, its range—increases because the projectile must "catch up" with the ground that is curving away from it. Trajectories B and C are of this type. The actual calculation of these trajectories is beyond the scope of this textbook, but you should be able to understand the factors that influence the trajectory.

If the launch speed v_0 is sufficiently large, there comes a point where the curve of the trajectory and the curve of the earth are parallel. In this case, the projectile "falls" but it never gets any closer to the ground! This is the situation for trajectory D. A closed trajectory around a planet or star, such as trajectory D, is called an **orbit.**

The most important point of this qualitative analysis is that **an orbiting projectile is in free fall.** This is, admittedly, a strange idea, but one worth careful thought. An orbiting projectile is really no different from a thrown baseball or a car driving off a cliff. The only force acting on it is gravity, but its tangential velocity is so large that the curvature of its trajectory matches the curvature of the earth. When this happens, the projectile "falls" under the influence of gravity but never gets any closer to the surface.

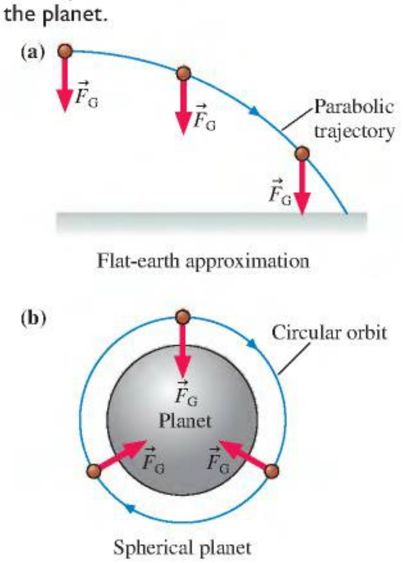


FIGURE 8.11 The "real" gravitational force is always directed toward the center of the planet.

In the flat-earth approximation, shown in **FIGURE 8.11a**, the gravitational force acting on an object of mass m is

$$\vec{F}_G = (mg, \text{vertically downward}) \qquad \text{(flat-earth approximation)} \tag{8.8}$$

But since stars and planets are actually spherical (or very close to it), the "real" force of gravity acting on an object is directed toward the *center* of the planet, as shown in **FIGURE 8.11b**. In this case the gravitational force is

$$\vec{F}_G = (mg, \text{toward center}) \qquad \text{(spherical planet)} \tag{8.9}$$

That is, gravity is a central force causing the centripetal acceleration of uniform circular motion. Thus the gravitational force causes the object in Figure 8.11b to have acceleration

$$\vec{a} = \frac{\vec{F}_{net}}{m} = (g, \text{toward center}) \tag{8.10}$$

An object moving in a circle of radius r at speed v_{orbit} will have this centripetal acceleration if

$$a_r = \frac{(v_{orbit})^2}{r} = g \tag{8.11}$$

That is, if an object moves parallel to the surface with the speed

$$v_{orbit} = \sqrt{rg} \tag{8.12}$$

then the free-fall acceleration is exactly the centripetal acceleration needed for a circular orbit of radius r. An object with any other speed will not follow a circular orbit.

The earth's radius is $r = R_e = 6.37 \times 10^6$ m. (A table of useful astronomical data is inside the back cover of this book.) The orbital speed of a projectile just skimming the surface of an airless, bald earth is

$$v_{orbit} = \sqrt{rg} = \sqrt{(6.37 \times 10^6 \text{ m})(9.80 \text{ m/s}^2)} = 7900 \text{ m/s} \approx 16{,}000 \text{ mph}$$

Even if there were no trees and mountains, a real projectile moving at this speed would burn up from the friction of air resistance.

Satellites

Suppose, however, that we launched the projectile from a tower of height $h = 230 \text{ mi} \approx 3.8 \times 10^5$ m, just above the earth's atmosphere. This is approximately the height of the International Space Station and other low-earth-orbit satellites. Note that $h \ll R_e$, so the radius of the orbit $r = R_e + h = 6.75 \times 10^6$ m is only 5% greater than the earth's radius. Many people have a mental image that satellites orbit far above the earth, but in fact many satellites come pretty close to skimming the surface. Our calculation of v_{orbit} thus turns out to be quite a good estimate of the speed of a satellite in low earth orbit.

We can use v_{orbit} to calculate the period of a satellite orbit:

$$T = \frac{2\pi r}{v_{orbit}} = 2\pi\sqrt{\frac{r}{g}} \qquad (8.13)$$

The International Space Station is in free fall.

For a low earth orbit, with $r = R_e + 230$ miles, we find $T = 5210 \text{ s} = 87$ min. The period of the International Space Station at an altitude of 230 mi is, indeed, close to 87 minutes. (The actual period is 93 min. The difference, you'll learn in Chapter 13, arises because g is slightly less at a satellite's altitude.)

When we discussed *weightlessness* in Chapter 6, we discovered that it occurs during free fall. We asked the question, at the end of « Section 6.3, whether astronauts and their spacecraft were in free fall. We can now give an affirmative answer: They are, indeed, in free fall. They are falling continuously around the earth, under the influence of only the gravitational force, but never getting any closer to the ground because the earth's surface curves beneath them. Weightlessness in space is no different from the weightlessness in a free-falling elevator. It does *not* occur from an absence of gravity. Instead, the astronaut, the spacecraft, and everything in it are weightless because they are all falling together.

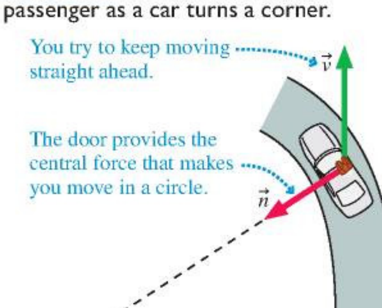
8.4 Reasoning About Circular Motion

Some aspects of circular motion are puzzling and counterintuitive. Examining a few of these will give us a chance to practice Newtonian reasoning.

Centrifugal Force?

If the car turns a corner quickly, you feel "thrown" against the door. But there's really no such force because there is no agent exerting it. FIGURE 8.12 shows a bird's-eye view of you riding in a car as it makes a left turn. You try to continue moving in a straight line, obeying Newton's first law, when—without having been provoked—the door suddenly turns in front of you and runs into you! You do, indeed, then feel the force of the door because it is now the normal force of the door, pointing *inward* toward the center of the curve, causing you to turn the corner. But you were not "thrown" into the door; the door ran into you.

FIGURE 8.12 Bird's-eye view of a passenger as a car turns a corner.



The "force" that seems to push an object to the outside of a circle is commonly known as the *centrifugal force*. Despite having a name, the centrifugal force is fictitious. It describes your experience *relative to a noninertial reference frame,* but there really is no such force. **You must always use Newton's laws in an inertial reference frame.** There are no centrifugal forces in an inertial reference frame.

NOTE You might wonder if the *rtz*-coordinate system is an inertial reference frame. It is. We're using the *rtz*-coordinates to establish directions for decomposing vectors, but we're not making measurements in the *rtz*-system. That is, velocities and accelerations are measured in the laboratory reference frame. The particle would always be at rest ($\vec{v} = \vec{0}$) if we measured velocities in a reference frame attached to the particle.

Gravity on a Rotating Earth

There is one small problem with the admonition that you must use Newton's laws in an inertial reference frame: A reference frame attached to the ground isn't truly inertial because of the earth's rotation. Fortunately, we can make a simple correction that allows us to continue using Newton's laws on the earth's surface.

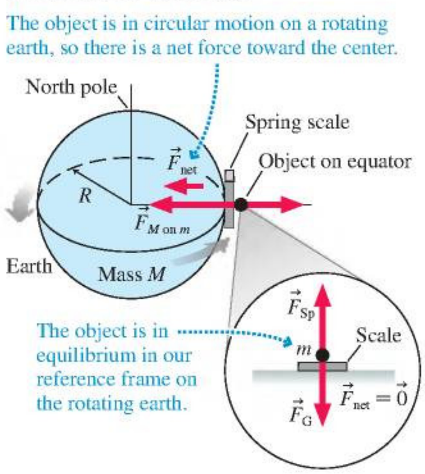


FIGURE 8.13 The earth's rotation affects the measured value of g.

FIGURE 8.13 shows an object being weighed by a spring scale on the earth's equator. An observer hovering above the north pole sees two forces on the object: the gravitational force $\vec{F}_{M\text{ on }m}$, given by Newton's law of gravity, and the outward spring force $\vec{F}_{Sp}$. The object moves in a circle as the earth rotates, so Newton's second law is

$$\sum F_r = F_{M\text{ on }m} - F_{Sp} = m\omega^2 R$$

where ω is the angular speed of the rotating earth. The spring-scale reading $F_{Sp} = F_{M\text{ on }m} - m\omega^2 R$ is *less* than it would be on a nonrotating earth.

The blow-up in Figure 8.13 shows how we see things in a noninertial, flat-earth reference frame. For us the object is at rest, in equilibrium, hence the upward spring force must be exactly balanced by a downward gravitational force $\vec{F}_G$. Thus $F_{Sp} = F_G$.

Now both we and the hovering, inertial observer see the same reading on the scale. If F_{Sp} is the same for both of us, then

$$F_G = F_{M\text{ on }m} - m\omega^2 R \tag{8.14}$$

In other words, force $\vec{F}_G$—what we called the *effective* gravitational force in Chapter 6—is slightly less than the true gravitational force $\vec{F}_{M\text{ on }m}$ because of the earth's rotation. In essence, $m\omega^2 R$ is the centrifugal force, a fictitious force trying—from our perspective in a noninertial reference frame—to "throw" us off the rotating platform. There really is no such force, but—this is the important point—**we can continue to use Newton's laws in our rotating reference frame if we pretend there is.**

Because $F_G = mg$ for an object at rest, the effect of the centrifugal term in Equation 8.14 is to make g a little smaller than it would be on a nonrotating earth:

$$g = \frac{F_G}{m} = \frac{F_{M\text{ on }m} - m\omega^2 R}{m} = \frac{GM}{R^2} - \omega^2 R = g_{earth} - \omega^2 R \tag{8.15}$$

We calculated $g_{earth} = 9.83\text{ m/s}^2$ in Chapter 6. Using $\omega = 1$ rev/day (which must be converted to SI units) and $R = 6370$ km, we find $\omega^2 R = 0.033\text{ m/s}^2$ at the equator. Thus the free-fall acceleration—what we actually measure in our rotating reference frame—is about 9.80 m/s^2, exactly what we measure in the laboratory.

Things are a little more complicated at other latitudes, but the bottom line is that we can safely use Newton's laws in our rotating, noninertial reference frame on the earth's surface if we calculate the gravitational force—as we've been doing—as $F_G = mg$ with g the measured free-fall value, a value that compensates for our rotation, rather than the purely gravitational g_{earth}.

Why Does the Water Stay in the Bucket?

If you swing a bucket of water over your head quickly, the water stays in, but you'll get a shower if you swing too slowly. Why? We'll answer this question by starting with an equivalent situation, a roller coaster doing a loop-the-loop.

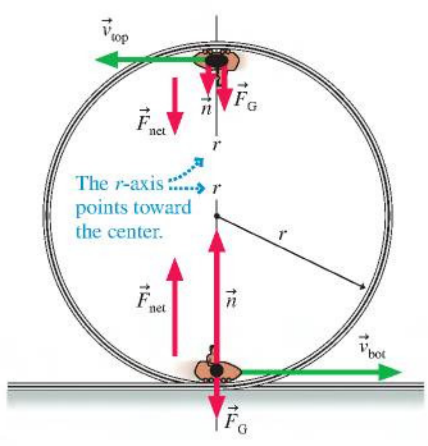


FIGURE 8.14 A roller-coaster car going around a loop-the-loop.

FIGURE 8.14 shows a roller-coaster car going around a vertical loop-the-loop of radius r. Why doesn't the car fall off at the top of the circle? Now, motion in a vertical circle is *not* uniform circular motion; the car slows down as it goes up one side and speeds up as it comes back down the other. But at the very top and very bottom points, only the car's direction is changing, not its speed, so at those points the acceleration is purely centripetal. Thus **there must be a net force toward the center of the circle.**

First consider the very bottom of the loop. To have a net force toward the center—upward at this point—requires $n > F_G$. The normal force has to *exceed* the gravitational force to provide the net force needed to "turn the corner" at the bottom of the circle. This is why you "feel heavy" at the bottom of the circle or at the bottom of a valley on a roller coaster.

We can analyze the situation quantitatively by writing the r-component of Newton's second law. At the bottom of the circle, with the r-axis pointing upward, we have

$$\sum F_r = n_r + (F_G)_r = n - mg = ma_r = \frac{m(v_{bot})^2}{r} \tag{8.16}$$

From Equation 8.16 we find

$$n = mg + \frac{m(v_{bot})^2}{r} \tag{8.17}$$

The normal force at the bottom is *larger* than mg.

Things are a little trickier as the roller-coaster car crosses the top of the loop. Whereas the normal force of the track pushes up when the car is at the bottom of the circle, it *presses down* when the car is at the top and the track is above the car. Think about the free-body diagram to make sure you agree.

The car is still moving in a circle, so there *must* be a net force toward the center of the circle. The r-axis, which points toward the center of the circle, now points *downward*. Consequently, both forces have *positive* components. Newton's second law at the top of the circle is

$$\sum F_r = n_r + (F_G)_r = n + mg = \frac{m(v_{top})^2}{r} \tag{8.18}$$

Thus at the top the normal force of the track on the car is

$$n = \frac{m(v_{top})^2}{r} - mg \tag{8.19}$$

The normal force at the top can exceed mg if v_{top} is large enough. Our interest, however, is in what happens as the car goes slower and slower. As v_{top} decreases, there comes a point when n reaches zero. "No normal force" means "no contact," so at that speed, the track is *not* pushing against the car. Instead, the car is able to complete the circle because gravity alone provides sufficient centripetal acceleration.

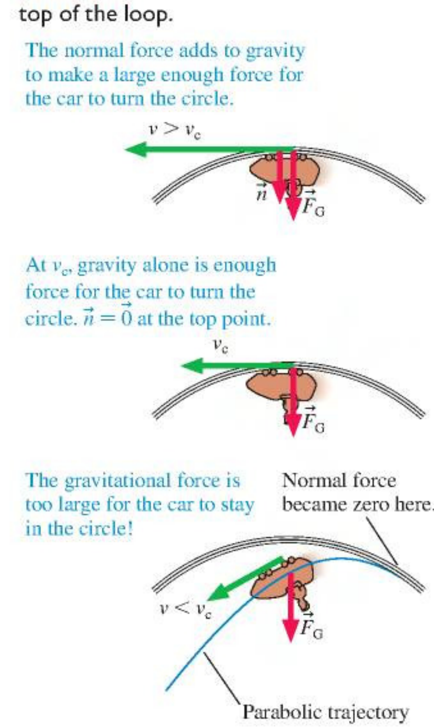
The speed at which $n = 0$ is called the *critical speed* v_c:

$$v_c = \sqrt{\frac{rmg}{m}} = \sqrt{rg} \tag{8.20}$$

The critical speed is the slowest speed at which the car can complete the circle. Equation 8.19 would give a negative value for n if $v < v_c$, but that is physically impossible. The track can push against the wheels of the car ($n > 0$), but it can't pull on them. If $v < v_c$, the car cannot turn the full loop but, instead, comes off the track and becomes a projectile! FIGURE 8.15 summarizes our reasoning.

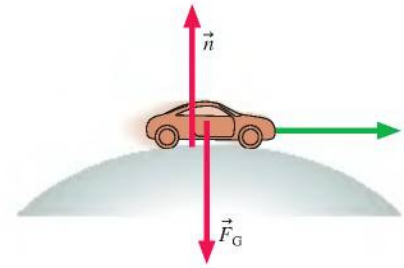
Water stays in a bucket swung over your head for the same reason: Circular motion requires a net force toward the center of the circle. At the top of the circle—if you swing the bucket fast enough—the bucket adds to the force of gravity by pushing *down* on the water, just like the downward normal force of the track on the roller-coaster car. As long as the bucket is pushing against the water, the bucket and the water are in contact and thus the water is "in" the bucket. As you swing slower and slower, requiring the water to have less and less centripetal acceleration, the bucket-on-water normal force decreases until it becomes zero at the critical speed. At the critical speed, gravity alone provides sufficient centripetal acceleration. Below the critical speed, gravity provides *too much* downward force for circular motion, so the water leaves the bucket and becomes a projectile following a parabolic trajectory toward your head!

FIGURE 8.15 A roller-coaster car at the top of the loop.



STOP TO THINK 8.3 An out-of-gas car is rolling over the top of a hill at speed v. At this instant,

a. $n > F_G$
b. $n < F_G$
c. $n = F_G$
d. We can't tell about n without knowing v.



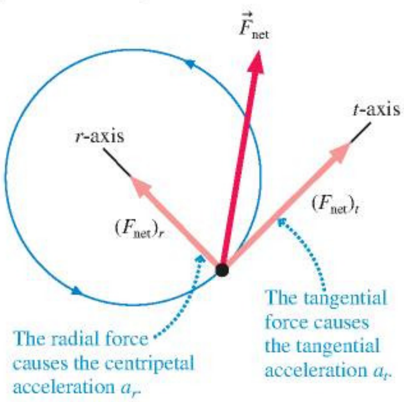
8.5 Nonuniform Circular Motion

Many interesting examples of circular motion involve objects whose speed changes. As we've already noted, a roller-coaster car doing a loop-the-loop slows down as it goes up one side, speeds up as it comes back down the other side. Circular motion with a changing speed is called *nonuniform circular motion*.

FIGURE 8.16 shows a particle moving in a circle of radius r. In addition to a radial force component—required for all circular motion—this particle experiences a *tangential* force component $(F_{net})_t$ and hence a tangential acceleration

$$a_t = \frac{dv_t}{dt} \tag{8.21}$$

FIGURE 8.16 Net force $\vec{F}_{net}$ is applied to a particle moving in a circle.



Now v_t is the particle's velocity *around* the circle, with speed $v = |v_t|$, so a tangential force component causes the particle to change speed. That is, the particle is undergoing nonuniform circular motion. Note that $(F_{net})_t$, like v_t, is positive when ccw, negative when cw.

Force and acceleration are still related to each other through Newton's second law:

$$\begin{aligned}(F_{net})_r &= \sum F_r = ma_r = \frac{mv_t^2}{r} = m\omega^2 r\\ (F_{net})_t &= \sum F_t = ma_t\\ (F_{net})_z &= \sum F_z = 0\end{aligned} \tag{8.22}$$

If the tangential force is constant, you can apply what you know about constant-acceleration kinematics to solve for v_t at a later time.

NOTE Equations 8.22 differ from Equations 8.7 for uniform circular motion only in the fact that a_t is no longer zero.

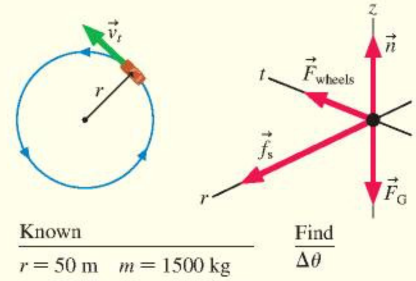
EXAMPLE 8.6 Sliding out of the curve

A 1500 kg car drives around a flat, 50-m-diameter track, starting from rest. The drive wheels supply a small but steady 525 N force in the forward direction. The coefficient of static friction between the car tires and the road is 0.90. How many revolutions of the track have been made when the car slides out of the curve?

MODEL Model the car as a particle in nonuniform circular motion. Assume that rolling friction and air resistance can be neglected.

VISUALIZE **FIGURE 8.17** shows a pictorial representation. As in earlier examples, it's static friction, perpendicular to the tires, that causes the centripetal acceleration of circular motion. The propulsion force is a tangential force. For the first time, we need a free-body diagram showing forces in three dimensions.

FIGURE 8.17 Pictorial representation of a car speeding up around a circle.



SOLVE At slow speeds, static friction in the radial direction keeps the car moving in a circle. But there's an upper limit to the size of the static friction force, and the car will begin to slide out of the curve when that limit is reached. The r-component of Newton's second law is

$$\sum F_r = f_s = \frac{mv_t^2}{r}$$

That is, static friction increases proportional to v_t^2 until the car reaches a velocity v_{max} at which the static friction is $f_{s\,max}$.

Recall that the maximum possible static friction is $f_{s\,max} = \mu_s n$. We can find the normal force from the z-component of Newton's second law:

$$\sum F_z = n - F_G = 0$$

Thus $n = F_G = mg$ and $f_{s\,max} = \mu_s mg$. Combining these two equations, we see that the mass cancels and we have

$$v_{max}^2 = \mu_s rg$$

How far does the car have to travel to reach this speed? We can find the car's tangential acceleration from the t-component of the second law: $a_t = F_{wheels}/m$. This is a constant acceleration, so we can use constant-acceleration kinematics. Let s measure the distance around the circle—the arc length. Thus, because the initial velocity is $v_0 = 0$, we have

$$v_t^2 = v_0^2 + 2a_s s = 2a_s s = \frac{2sF_{wheels}}{m}$$

You'll recall that the angular displacement, measured in radians, is $\Delta\theta = s/r$. So when the car reaches velocity v_t, it has revolved through an angle

$$\Delta\theta = \frac{s}{r} = \frac{mv_t^2}{2rF_{wheels}}$$

Using the maximum speed before sliding, we find that the car slides out of the curve after revolving through an angle

$$\Delta\theta_{max} = \frac{mv_{max}^2}{2rF_{wheels}} = \frac{m}{2rF_{wheels}} \times \mu_s rg = \frac{\mu_s mg}{2F_{wheels}}$$

For the car in this problem,

$$\Delta\theta_{max} = \frac{(0.90)(1500\text{ kg})(9.80\text{ m/s}^2)}{2(525\text{ N})}$$

$$= 12.6\text{ rad} \times \frac{1\text{ rev}}{2\pi\text{ rad}} = 2.0\text{ rev}$$

It completes 2.0 revolutions before it starts to slide.

ASSESS A 525 N force on a 1500 kg car causes a tangential acceleration of $a_t \approx 0.3\text{ m/s}^2$. That's a quite modest acceleration, so it seems reasonable that the car would complete 2 rev before gaining enough speed to start sliding.

We've come a long way since our first dynamics problems in Chapter 6, but our basic strategy has not changed.

PROBLEM-SOLVING STRATEGY 8.1 MP

Circular-motion problems

MODEL Model the object as a particle and make other simplifying assumptions.

VISUALIZE Draw a pictorial representation. Use rtz-coordinates.

- Establish a coordinate system with the r-axis pointing toward the center of the circle.
- Show important points in the motion on a sketch. Define symbols and identify what the problem is trying to find.
- Identify the forces and show them on a free-body diagram.

SOLVE Newton's second law is

$$(F_{net})_r = \sum F_r = ma_r = \frac{mv_t^2}{r} = m\omega^2 r$$

$$(F_{net})_t = \sum F_t = ma_t$$

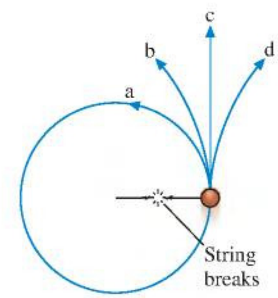
$$(F_{net})_z = \sum F_z = 0$$

- Determine the force components from the free-body diagram. Be careful with signs.
- The tangential acceleration for uniform circular motion is $a_t = 0$.
- Solve for the acceleration, then use kinematics to find velocities and positions.

ASSESS Check that your result has the correct units and significant figures, is reasonable, and answers the question.

Exercise 11

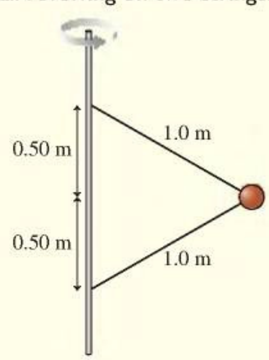
STOP TO THINK 8.4 A ball on a string is swung in a vertical circle. The string happens to break when it is parallel to the ground and the ball is moving up. Which trajectory does the ball follow?



CHALLENGE EXAMPLE 8.7 | Swinging on two strings

The 250 g ball shown in FIGURE 8.18 revolves in a horizontal plane as the vertical shaft spins. What is the critical angular speed, in rpm, that the shaft must exceed to keep both strings taut?

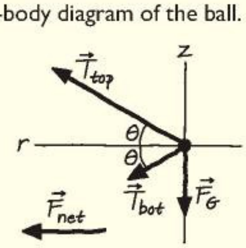
FIGURE 8.18 A ball revolving on two strings.



MODEL Model the ball as a particle in uniform circular motion. For both strings to be straight, as shown, both must be under tension. If the angular speed is slowly decreased, eventually the lower string will go slack and the ball will sag. The critical angular speed ω_c is the angular speed at which the tension in the lower string reaches zero. We need to find an expression for the tension in the lower string, then determine when that tension becomes zero.

VISUALIZE FIGURE 8.19 is the ball's free-body diagram with the r-axis pointing toward the center of the circle. The ball is acted on by two tension forces, at equal angles above and below horizontal, and by gravity. The free-body diagram is similar to Example 8.5, the rock in the sling, but with an additional tension force.

FIGURE 8.19 Free-body diagram of the ball.



SOLVE This is uniform circular motion, so we need to consider only the r- and z-components of Newton's second law. All the information is on the free-body diagram, where we see that gravity has only a z-component but the tensions have both r- and z-components. The two equations are

$$\sum F_r = T_{top}\cos\theta + T_{bot}\cos\theta = m\omega^2 r$$
$$\sum F_z = T_{top}\sin\theta - T_{bot}\sin\theta - mg = 0$$

Factoring out the $\cos\theta$ and $\sin\theta$ terms, we have two simultaneous equations:

$$T_{top} + T_{bot} = \frac{m\omega^2 r}{\cos\theta}$$
$$T_{top} - T_{bot} = \frac{mg}{\sin\theta}$$

Subtracting the second equation from the first will eliminate T_{top}:

$$2T_{bot} = \frac{m\omega^2 r}{\cos\theta} - \frac{mg}{\sin\theta}$$

and thus

$$T_{bot} = \frac{m}{2}\left(\frac{\omega^2 r}{\cos\theta} - \frac{g}{\sin\theta}\right)$$

You can see that this expression becomes negative—a physically impossible situation—if ω is too small. The angular speed at which the tension reaches zero—the critical angular speed—is found by setting the expression in parentheses equal to zero. This gives

$$\omega_c = \sqrt{\frac{g}{r\tan\theta}}$$

For our situation,

$$r = \sqrt{(1.0\text{ m})^2 - (0.50\text{ m})^2} = 0.866\text{ m}$$
$$\theta = \sin^{-1}[(0.50\text{ m})/(1.0\text{ m})] = 30°$$

Thus the critical angular speed is

$$\omega_c = \sqrt{\frac{9.80\text{ m/s}^2}{(0.866\text{ m})\tan 30°}} = 4.40\text{ rad/s}$$

Converting to rpm:

$$\omega_c = 4.40\text{ rad/s} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}} = 42\text{ rpm}$$

ASSESS ω_c is the *minimum* angular speed needed to keep both strings taut. For a ball attached to meter-long strings, 42 rpm—a bit less than 1 revolution per second—seems plausible. Your intuition probably suggests that the bottom string wouldn't be taut if the shaft spun at only a few rpm, and hundreds of rpm seems much too high. Remember that the goal of Assess is not to prove that an answer is correct but to rule out answers that, with a little thought, are clearly wrong.

SUMMARY

The goal of Chapter 8 has been to learn to solve problems about motion in two dimensions.

GENERAL PRINCIPLES

Newton's Second Law

Expressed in x- and y-component form:

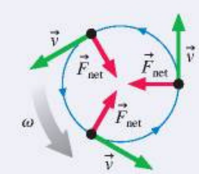
$$(F_{net})_x = \sum F_x = ma_x$$
$$(F_{net})_y = \sum F_y = ma_y$$

Expressed in rtz-component form:

$$(F_{net})_r = \sum F_r = ma_r = \frac{mv_t^2}{r} = m\omega^2 r$$
$$(F_{net})_t = \sum F_t = \begin{cases} 0 & \text{uniform circular motion} \\ ma_t & \text{nonuniform circular motion} \end{cases}$$
$$(F_{net})_z = \sum F_z = 0$$

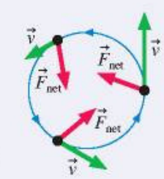
Uniform Circular Motion

- Speed is constant.
- $\vec{F}_{net}$ points toward the center of the circle.
- The **centripetal acceleration** $\vec{a}$ points toward the center of the circle. It changes the particle's direction but not its speed.



Nonuniform Circular Motion

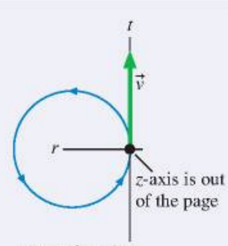
- Speed changes.
- $\vec{F}_{net}$ and $\vec{a}$ have both radial and tangential components.
- The radial component changes the particle's direction.
- The tangential component changes the particle's speed.



IMPORTANT CONCEPTS

rtz-coordinates

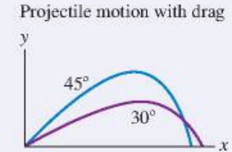
- The r-axis points toward the center of the circle.
- The t-axis is tangent, pointing counterclockwise.



Projectile motion

- With no drag, the x- and y-components of acceleration are independent. The trajectory is a parabola.
- With drag, the trajectory is not a parabola. Maximum range is achieved for an angle less than 45°.

Projectile motion with drag



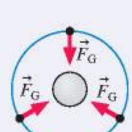
APPLICATIONS

Orbits

An object acted on only by gravity has a circular orbit of radius r if its speed is

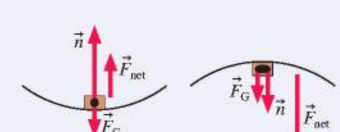
$$v = \sqrt{rg}$$

The object is in free fall.



Circular motion on surfaces

Circular motion requires a net force pointing to the center. n must be > 0 for the object to be in contact with a surface.



TERMS AND NOTATION

rtz-coordinate system
central force
central-force model
orbit

CONCEPTUAL QUESTIONS

1. In uniform circular motion, which of the following are constant: speed, velocity, angular velocity, centripetal acceleration, magnitude of the net force?
2. A car runs out of gas while driving down a hill. It rolls through the valley and starts up the other side. At the very bottom of the valley, which of the free-body diagrams in FIGURE Q8.2 is correct? The car is moving to the right, and drag and rolling friction are negligible.

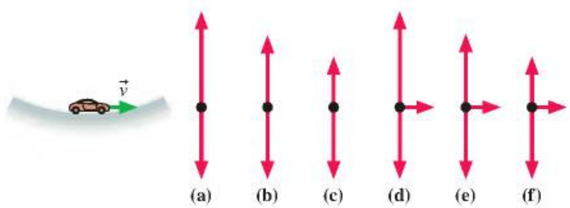


FIGURE Q8.2

3. FIGURE Q8.3 is a bird's-eye view of particles on strings moving in horizontal circles on a tabletop. All are moving at the same speed. Rank in order, from largest to smallest, the tensions T_a to T_d. Give your answer in the form a > b = c > d and explain your ranking.

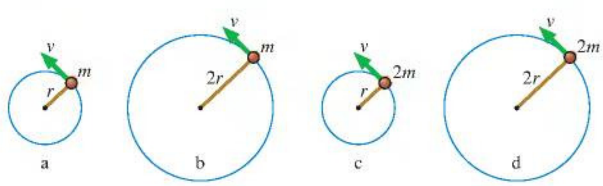


FIGURE Q8.3

4. Tarzan swings through the jungle on a massless vine. At the lowest point of his swing, is the tension in the vine greater than, less than, or equal to the gravitational force on Tarzan? Explain.
5. FIGURE Q8.5 shows two balls of equal mass moving in vertical circles. Is the tension in string A greater than, less than, or equal to the tension in string B if the balls travel over the top of the circle (a) with equal speed and (b) with equal angular velocity?

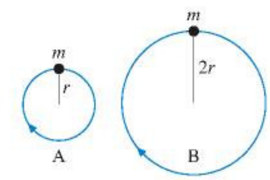


FIGURE Q8.5

6. Ramon and Sally are observing a toy car speed up as it goes around a circular track. Ramon says, "The car's speeding up, so there must be a net force parallel to the track." "I don't think so," replies Sally. "It's moving in a circle, and that requires centripetal acceleration. The net force has to point to the center of the circle." Do you agree with Ramon, Sally, or neither? Explain.
7. A jet plane is flying on a level course at constant speed. The engines are at full throttle.
   a. What is the net force on the plane? Explain.
   b. Draw a free-body diagram of the plane as seen from the side with the plane flying to the right. Name (don't just label) any and all forces shown on your diagram.
   c. Airplanes bank when they turn. Draw a free-body diagram of the plane as seen from behind as it makes a right turn.
   d. *Why* do planes bank as they turn? Explain.
8. A small projectile is launched parallel to the ground at height $h = 1$ m with sufficient speed to orbit a completely smooth, airless planet. A bug rides inside a small hole inside the projectile. Is the bug weightless? Explain.
9. You can swing a ball on a string in a vertical circle if you swing it fast enough. But if you swing too slowly, the string goes slack as the ball nears the top. Explain *why* there's a minimum speed to keep the ball moving in a circle.
10. A golfer starts with the club over her head and swings it to reach maximum speed as it contacts the ball. Halfway through her swing, when the golf club is parallel to the ground, does the acceleration vector of the club head point (a) straight down, (b) parallel to the ground, approximately toward the golfer's shoulders, (c) approximately toward the golfer's feet, or (d) toward a point above the golfer's head? Explain.

EXERCISES AND PROBLEMS

Problems labeled integrate material from earlier chapters.

Exercises

Section 8.1 Dynamics in Two Dimensions

1. ‖ As a science fair project, you want to launch an 800 g model rocket straight up and hit a horizontally moving target as it passes 30 m above the launch point. The rocket engine provides a constant thrust of 15.0 N. The target is approaching at a speed of 15 m/s. At what horizontal distance between the target and the rocket should you launch?
2. ‖ A 500 g model rocket is on a cart that is rolling to the right at a speed of 3.0 m/s. The rocket engine, when it is fired, exerts an 8.0 N vertical thrust on the rocket. Your goal is to have the rocket pass through a small horizontal hoop that is 20 m above the ground. At what horizontal distance left of the hoop should you launch?
3. ‖ A 4.0×10^{10} kg asteroid is heading directly toward the center of the earth at a steady 20 km/s. To save the planet, astronauts strap a giant rocket to the asteroid perpendicular to its direction of travel. The rocket generates 5.0×10^9 N of thrust. The rocket is fired when the asteroid is 4.0×10^6 km away from earth. You can ignore the earth's gravitational force on the asteroid and their rotation about the sun.

a. If the mission fails, how many hours is it until the asteroid impacts the earth?
b. The radius of the earth is 6400 km. By what minimum angle must the asteroid be deflected to just miss the earth?
c. What is the actual angle of deflection if the rocket fires at full thrust for 300 s before running out of fuel?

4. || A 55 kg astronaut who weighs 180 N on a distant planet is pondering whether she can leap over a 3.5-m-wide chasm without falling in. If she leaps at a 15° angle, what initial speed does she need to clear the chasm?

Section 8.2 Uniform Circular Motion

5. | A 1500 kg car drives around a flat 200-m-diameter circular track at 25 m/s. What are the magnitude and direction of the net force on the car? What causes this force?
6. | A 1500 kg car takes a 50-m-radius unbanked curve at 15 m/s. What is the size of the friction force on the car?
7. || A 200 g block on a 50-cm-long string swings in a circle on a horizontal, frictionless table at 75 rpm.
   a. What is the speed of the block?
   b. What is the tension in the string?
8. || In the Bohr model of the hydrogen atom, an electron (mass $m = 9.1 \times 10^{-31}$ kg) orbits a proton at a distance of 5.3×10^{-11} m. The proton pulls on the electron with an electric force of 8.2×10^{-8} N. How many revolutions per second does the electron make?
9. | Suppose the moon were held in its orbit not by gravity but by a massless cable attached to the center of the earth. What would be the tension in the cable? Use the table of astronomical data inside the back cover of the book.
10. || A highway curve of radius 500 m is designed for traffic moving at a speed of 90 km/h. What is the correct banking angle of the road?
11. || It is proposed that future space stations create an artificial gravity by rotating. Suppose a space station is constructed as a 1000-m-diameter cylinder that rotates about its axis. The inside surface is the deck of the space station. What rotation period will provide "normal" gravity?
12. || A 5.0 g coin is placed 15 cm from the center of a turntable. The coin has static and kinetic coefficients of friction with the turntable surface of $\mu_s = 0.80$ and $\mu_k = 0.50$. The turntable very slowly speeds up to 60 rpm. Does the coin slide off?
13. || Mass m_1 on the frictionless table of **FIGURE EX8.13** is connected by a string through a hole in the table to a hanging mass m_2. With what speed must m_1 rotate in a circle of radius r if m_2 is to remain hanging at rest?

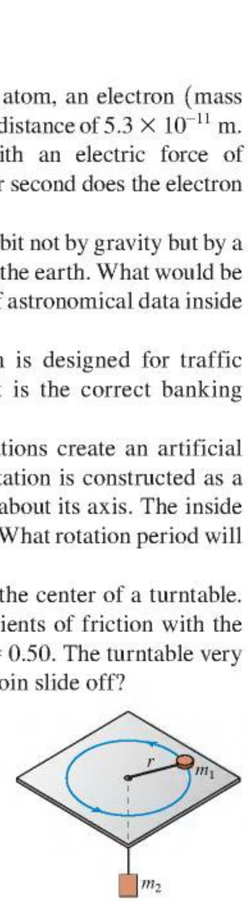


FIGURE EX8.13

Section 8.3 Circular Orbits

14. | A satellite orbiting the moon very near the surface has a period of 110 min. What is free-fall acceleration on the surface of the moon?
15. || What is free-fall acceleration toward the sun at the distance of the earth's orbit? Astronomical data are inside the back cover of the book.
16. || A 9.4×10^{21} kg moon orbits a distant planet in a circular orbit of radius 1.5×10^{8} m. It experiences a 1.1×10^{19} N gravitational pull from the planet. What is the moon's orbital period in earth days?
17. || Communications satellites are placed in circular orbits where they stay directly over a fixed point on the equator as the earth rotates. These are called *geosynchronous orbits*. The altitude of a geosynchronous orbit is 3.58×10^{7} m ($\approx$22,000 miles).
   a. What is the period of a satellite in a geosynchronous orbit?
   b. Find the value of g at this altitude.
   c. What is the weight of a 2000 kg satellite in a geosynchronous orbit?

Section 8.4 Reasoning About Circular Motion

18. | A car drives over the top of a hill that has a radius of 50 m. What maximum speed can the car have at the top without flying off the road?
19. || The weight of passengers on a roller coaster increases by 50% as the car goes through a dip with a 30 m radius of curvature. What is the car's speed at the bottom of the dip?
20. || A roller coaster car crosses the top of a circular loop-the-loop at twice the critical speed. What is the ratio of the normal force to the gravitational force?
21. || The normal force equals the magnitude of the gravitational force as a roller coaster car crosses the top of a 40-m-diameter loop-the-loop. What is the car's speed at the top?
22. || A student has 65-cm-long arms. What is the minimum angular velocity (in rpm) for swinging a bucket of water in a vertical circle without spilling any? The distance from the handle to the bottom of the bucket is 35 cm.
23. | While at the county fair, you decide to ride the Ferris wheel. Having eaten too many candy apples and elephant ears, you find the motion somewhat unpleasant. To take your mind off your stomach, you wonder about the motion of the ride. You estimate the radius of the big wheel to be 15 m, and you use your watch to find that each loop around takes 25 s.
   a. What are your speed and the magnitude of your acceleration?
   b. What is the ratio of your weight at the top of the ride to your weight while standing on the ground?
   c. What is the ratio of your weight at the bottom of the ride to your weight while standing on the ground?
24. || A 500 g ball swings in a vertical circle at the end of a 1.5-m-long string. When the ball is at the bottom of the circle, the tension in the string is 15 N. What is the speed of the ball at that point?
25. || A 500 g ball moves in a vertical circle on a 102-cm-long string. If the speed at the top is 4.0 m/s, then the speed at the bottom will be 7.5 m/s. (You'll learn how to show this in Chapter 10.)
   a. What is the gravitational force acting on the ball?
   b. What is the tension in the string when the ball is at the top?
   c. What is the tension in the string when the ball is at the bottom?
26. || A heavy ball with a weight of 100 N ($m = 10.2$ kg) is hung from the ceiling of a lecture hall on a 4.5-m-long rope. The ball is pulled to one side and released to swing as a pendulum, reaching a speed of 5.5 m/s as it passes through the lowest point. What is the tension in the rope at that point?

Section 8.5 Nonuniform Circular Motion

27. || A toy train rolls around a horizontal 1.0-m-diameter track. The coefficient of rolling friction is 0.10. How long does it take the train to stop if it's released with an angular speed of 30 rpm?
28. || A new car is tested on a 200-m-diameter track. If the car speeds up at a steady 1.5 m/s^2, how long after starting is the magnitude of its centripetal acceleration equal to the tangential acceleration?

29. || An 85,000 kg stunt plane performs a loop-the-loop, flying in a 260-m-diameter vertical circle. At the point where the plane is flying straight down, its speed is 55 m/s and it is speeding up at a rate of 12 m/s per second.
   a. What is the magnitude of the net force on the plane? You can neglect air resistance.
   b. What angle does the net force make with the horizontal? Let an angle above horizontal be positive and an angle below horizontal be negative.

30. || Three cars are driving at 25 m/s along the road shown in FIGURE EX8.30. Car B is at the bottom of a hill and car C is at the top. Both hills have a 200 m radius of curvature. Suppose each car suddenly brakes hard and starts to skid. What is the tangential acceleration (i.e., the acceleration parallel to the road) of each car? Assume $\mu_k = 1.0$.

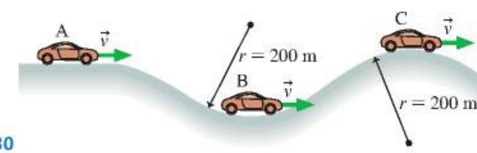


FIGURE EX8.30

Problems

31. || Derive Equations 8.3 for the acceleration of a projectile subject to drag.

32. || A 100 g bead slides along a frictionless wire with the parabolic
CALC shape $y = (2\text{ m}^{-1})x^2$.
   a. Find an expression for a_y, the vertical component of acceleration, in terms of x, v_x, and a_x.
   Hint: Use the basic definitions of velocity and acceleration.
   b. Suppose the bead is released at some negative value of x and has a speed of 2.3 m/s as it passes through the lowest point of the parabola. What is the net force on the bead at this instant? Write your answer in component form.

33. || Space scientists have a large test chamber from which all the air can be evacuated and in which they can create a horizontal uniform electric field. The electric field exerts a constant horizontal force on a charged object. A 15 g charged projectile is launched with a speed of 6.0 m/s at an angle 35° above the horizontal. It lands 2.9 m in front of the launcher. What is the magnitude of the electric force on the projectile?

34. || A 5000 kg interceptor rocket is launched at an angle of 44.7°. The thrust of the rocket motor is 140,700 N.
   a. Find an equation $y(x)$ that describes the rocket's trajectory.
   b. What is the shape of the trajectory?
   c. At what elevation does the rocket reach the speed of sound, 330 m/s?

35. || A motorcycle daredevil plans to ride up a 2.0-m-high, 20° ramp, sail across a 10-m-wide pool filled with hungry crocodiles, and land at ground level on the other side. He has done this stunt many times and approaches it with confidence. Unfortunately, the motorcycle engine dies just as he starts up the ramp. He is going 11 m/s at that instant, and the rolling friction of his rubber tires (coefficient 0.02) is not negligible. Does he survive, or does he become crocodile food? Justify your answer by calculating the distance he travels through the air after leaving the end of the ramp.

36. ||| A rocket-powered hockey puck has a thrust of 2.0 N and a total mass of 1.0 kg. It is released from rest on a frictionless table, 4.0 m from the edge of a 2.0 m drop. The front of the rocket is pointed directly toward the edge. How far does the puck land from the base of the table?

37. || A 500 g model rocket is resting horizontally at the top edge of a 40-m-high wall when it is accidentally bumped. The bump pushes it off the edge with a horizontal speed of 0.5 m/s and at the same time causes the engine to ignite. When the engine fires, it exerts a constant 20 N horizontal thrust away from the wall.
   a. How far from the base of the wall does the rocket land?
   b. Describe the rocket's trajectory as it travels to the ground.

38. || A 2.0 kg projectile with initial velocity $\vec{v} = 8.0\,\hat{\imath}$ m/s experi-
CALC ences the variable force $\vec{F} = -2.0t\,\hat{\imath} + 4.0t^2\,\hat{\jmath}$ N, where t is in s.
   a. What is the projectile's speed at $t = 2.0$ s?
   b. At what instant of time is the projectile moving parallel to the y-axis?

39. || A 75 kg man weighs himself at the north pole and at the equator. Which scale reading is higher? By how much? Assume the earth is spherical.

40. || A concrete highway curve of radius 70 m is banked at a 15° angle. What is the maximum speed with which a 1500 kg rubber-tired car can take this curve without sliding?

41. || a. An object of mass m swings in a horizontal circle on a string of length L that tilts downward at angle θ. Find an expression for the angular velocity ω.
   b. A student ties a 500 g rock to a 1.0-m-long string and swings it around her head in a horizontal circle. At what angular speed, in rpm, does the string tilt down at a 10° angle?

42. ||| You've taken your neighbor's young child to the carnival to ride the rides. She wants to ride The Rocket. Eight rocket-shaped cars hang by chains from the outside edge of a large steel disk. A vertical axle through the center of the ride turns the disk, causing the cars to revolve in a circle. You've just finished taking physics, so you decide to figure out the speed of the cars while you wait. You estimate that the disk is 5 m in diameter and the chains are 6 m long. The ride takes 10 s to reach full speed, then the cars swing out until the chains are 20° from vertical. What is the cars' speed?

43. || A 4.4-cm-diameter, 24 g plastic ball is attached to a 1.2-m-long string and swung in a vertical circle. The ball's speed is 6.1 m/s at the point where it is moving straight up. What is the magnitude of the net force on the ball? Air resistance is not negligible.

44. || A charged particle of mass m moving with speed v in a plane perpendicular to a magnetic field experiences a force $\vec{F} =$ (qvB, perpendicular to $\vec{v}$), where q is the amount of charge and B is the magnetic field strength. Because the force is always perpendicular to the particle's velocity, the particle undergoes uniform circular motion. Find an expression for the period of the motion. Gravity can be neglected.

45. ||| Two wires are tied to the 2.0 kg sphere shown in FIGURE P8.45. The sphere revolves in a horizontal circle at constant speed.
   a. For what speed is the tension the same in both wires?
   b. What is the tension?

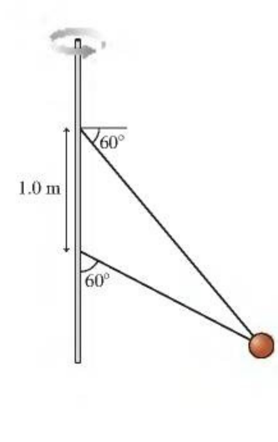


FIGURE P8.45

46. ||| Two wires are tied to the 300 g sphere shown in FIGURE P8.46. The sphere revolves in a horizontal circle at a constant speed of 7.5 m/s. What is the tension in each of the wires?

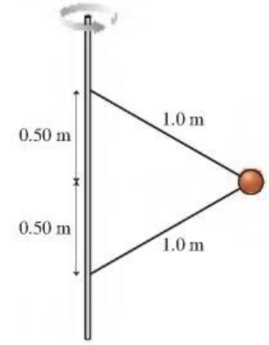


FIGURE P8.46

47. || A conical pendulum is formed by attaching a ball of mass m to a string of length L, then allowing the ball to move in a horizontal circle of radius r. FIGURE P8.47 shows that the string traces out the surface of a cone, hence the name.
   a. Find an expression for the tension T in the string.
   b. Find an expression for the ball's angular speed ω.
   c. What are the tension and angular speed (in rpm) for a 500 g ball swinging in a 20-cm-radius circle at the end of a 1.0-m-long string?

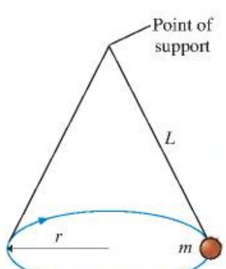


FIGURE P8.47

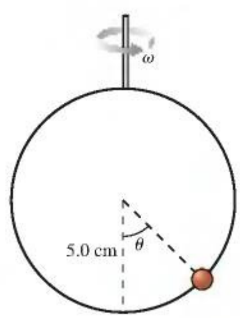


FIGURE P8.48

48. || The 10 mg bead in FIGURE P8.48 is free to slide on a frictionless wire loop. The loop rotates about a vertical axis with angular velocity ω. If ω is less than some critical value ω_c, the bead sits at the bottom of the spinning loop. When $\omega > \omega_c$, the bead moves out to some angle θ.
   a. What is ω_c in rpm for the loop shown in the figure?
   b. At what value of ω, in rpm, is $\theta = 30°$?

49. ||| In an old-fashioned amusement park ride, passengers stand inside a 5.0-m-diameter hollow steel cylinder with their backs against the wall. The cylinder begins to rotate about a vertical axis. Then the floor on which the passengers are standing suddenly drops away! If all goes well, the passengers will "stick" to the wall and not slide. Clothing has a static coefficient of friction against steel in the range 0.60 to 1.0 and a kinetic coefficient in the range 0.40 to 0.70. A sign next to the entrance says "No children under 30 kg allowed." What is the minimum angular speed, in rpm, for which the ride is safe?

50. || The ultracentrifuge is an important tool for separating and analyzing proteins. Because of the enormous centripetal accelerations, the centrifuge must be carefully balanced, with each sample matched by a sample of identical mass on the opposite side. Any difference in the masses of opposing samples creates a net force on the shaft of the rotor, potentially leading to a catastrophic failure of the apparatus. Suppose a scientist makes a slight error in sample preparation and one sample has a mass 10 mg larger than the opposing sample. If the samples are 12 cm from the axis of the rotor and the ultracentrifuge spins at 70,000 rpm, what is the magnitude of the net force on the rotor due to the unbalanced samples?
BIO

51. || In an amusement park ride called The Roundup, passengers stand inside a 16-m-diameter rotating ring. After the ring has acquired sufficient speed, it tilts into a vertical plane, as shown in FIGURE P8.51.
   a. Suppose the ring rotates once every 4.5 s. If a rider's mass is 55 kg, with how much force does the ring push on her at the top of the ride? At the bottom?
   b. What is the longest rotation period of the wheel that will prevent the riders from falling off at the top?

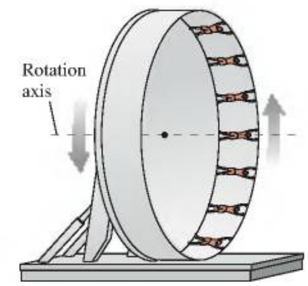


FIGURE P8.51

52. || Suppose you swing a ball of mass m in a vertical circle on a string of length L. As you probably know from experience, there is a minimum angular velocity ω_{min} you must maintain if you want the ball to complete the full circle without the string going slack at the top.
   a. Find an expression for ω_{min}.
   b. Evaluate ω_{min} in rpm for a 65 g ball tied to a 1.0-m-long string.

53. || A 30 g ball rolls around a 40-cm-diameter L-shaped track, shown in FIGURE P8.53, at 60 rpm. What is the magnitude of the *net* force that the track exerts on the ball? Rolling friction can be neglected.
Hint: The track exerts more than one force on the ball.

FIGURE P8.53

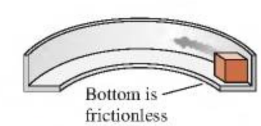


FIGURE P8.54

54. ||| FIGURE P8.54 shows a small block of mass m sliding around the inside of an L-shaped track of radius r. The bottom of the track is frictionless; the coefficient of kinetic friction between the block and the wall of the track is μ_k. The block's speed is v_0 at $t_0 = 0$. Find an expression for the block's speed at a later time t.

55. || The physics of circular motion sets an upper limit to the speed of human walking. (If you need to go faster, your gait changes from a walk to a run.) If you take a few steps and watch what's happening, you'll see that your body pivots in circular motion over your forward foot as you bring your rear foot forward for the next step. As you do so, the normal force of the ground on your foot decreases and your body tries to "lift off" from the ground.
BIO
   a. A person's center of mass is very near the hips, at the top of the legs. Model a person as a particle of mass m at the top of a leg of length L. Find an expression for the person's maximum walking speed v_{max}.
   b. Evaluate your expression for the maximum walking speed of a 70 kg person with a typical leg length of 70 cm. Give your answer in both m/s and mph, then comment, based on your experience, as to whether this is a reasonable result. A "normal" walking speed is about 3 mph.

56. ‖ A 100 g ball on a 60-cm-long string is swung in a vertical circle about a point 200 cm above the floor. The tension in the string when the ball is at the very bottom of the circle is 5.0 N. A very sharp knife is suddenly inserted, as shown in **FIGURE P8.56**, to cut the string directly below the point of support. How far to the right of where the string was cut does the ball hit the floor?

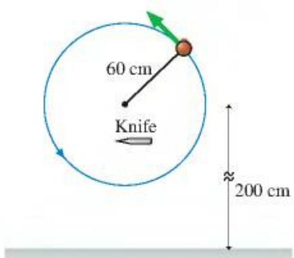


FIGURE P8.56

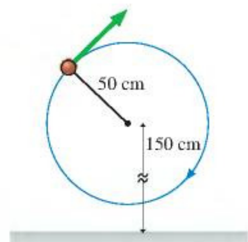


FIGURE P8.57

57. ‖ A 60 g ball is tied to the end of a 50-cm-long string and swung in a vertical circle. The center of the circle, as shown in **FIGURE P8.57**, is 150 cm above the floor. The ball is swung at the minimum speed necessary to make it over the top without the string going slack. If the string is released at the instant the ball is at the top of the loop, how far to the right does the ball hit the ground?

58. ‖ Elm Street has a pronounced dip at the bottom of a steep hill before going back uphill on the other side. Your science teacher has asked everyone in the class to measure the radius of curvature of the dip. Some of your classmates are using surveying equipment, but you decide to base your measurement on what you've learned in physics. To do so, you sit on a spring scale, drive through the dip at different speeds, and for each speed record the scale's reading as you pass through the bottom of the dip. Your data are as follows:

Speed (m/s)	Scale reading (N)
5	599
10	625
15	674
20	756
25	834

Sitting on the scale while the car is parked gives a reading of 588 N. Analyze your data, using a graph, to determine the dip's radius of curvature.

59. ‖ A 100 g ball on a 60-cm-long string is swung in a vertical circle about a point 200 cm above the floor. The string suddenly breaks when it is parallel to the ground and the ball is moving upward. The ball reaches a height 600 cm above the floor. What was the tension in the string an instant before it broke?

60. ‖ CALC Scientists design a new particle accelerator in which protons (mass 1.7×10^{-27} kg) follow a circular trajectory given by $\vec{r} = c\cos(kt^2)\hat{\imath} + c\sin(kt^2)\hat{\jmath}$, where $c = 5.0$ m and $k = 8.0 \times 10^4$ rad/s^2 are constants and t is the elapsed time.
a. What is the radius of the circle?
b. What is the proton's speed at $t = 3.0$ s?
c. What is the force on the proton at $t = 3.0$ s? Give your answer in component form.

61. ||| A 1500 kg car starts from rest and drives around a flat 50-m-diameter circular track. The forward force provided by the car's drive wheels is a constant 1000 N.
a. What are the magnitude and direction of the car's acceleration at $t = 10$ s? Give the direction as an angle from the r-axis.
b. If the car has rubber tires and the track is concrete, at what time does the car begin to slide out of the circle?

62. ||| A 500 g steel block rotates on a steel table while attached to a 2.0-m-long massless rod. Compressed air fed through the rod is ejected from a nozzle on the back of the block, exerting a thrust force of 3.5 N. The nozzle is 70° from the radial line, as shown in **FIGURE P8.62**. The block starts from rest.
a. What is the block's angular velocity after 10 rev?
b. What is the tension in the rod after 10 rev?

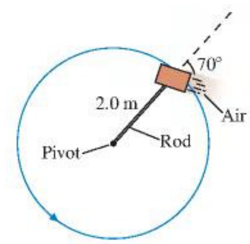


FIGURE P8.62

63. ||| A 2.0 kg ball swings in a vertical circle on the end of an 80-cm-long string. The tension in the string is 20 N when its angle from the highest point on the circle is $\theta = 30°$.
a. What is the ball's speed when $\theta = 30°$?
b. What are the magnitude and direction of the ball's acceleration when $\theta = 30°$?

In Problems 64 and 65 you are given the equation used to solve a problem. For each of these, you are to
a. Write a realistic problem for which this is the correct equation. Be sure that the answer your problem requests is consistent with the equation given.
b. Finish the solution of the problem.

64. $60 \text{ N} = (0.30 \text{ kg})\omega^2(0.50 \text{ m})$

65. $(1500 \text{ kg})(9.8 \text{ m/s}^2) - 11{,}760 \text{ N} = (1500 \text{ kg})\, v^2/(200 \text{ m})$

Challenge Problems

66. ||| Sam (75 kg) takes off up a 50-m-high, 10° frictionless slope on his jet-powered skis. The skis have a thrust of 200 N. He keeps his skis tilted at 10° after becoming airborne, as shown in **FIGURE CP8.66**. How far does Sam land from the base of the cliff?

FIGURE CP8.66

67. ||| CALC In the absence of air resistance, a projectile that lands at the elevation from which it was launched achieves maximum range when launched at a 45° angle. Suppose a projectile of mass m is launched with speed v_0 into a headwind that exerts a constant, horizontal retarding force $\vec{F}_{wind} = -F_{wind}\hat{\imath}$.
a. Find an expression for the angle at which the range is maximum.
b. By what percentage is the maximum range of a 0.50 kg ball reduced if $F_{wind} = 0.60$ N?

68. ||| The father of Example 8.2 stands at the summit of a conical hill as he spins his 20 kg child around on a 5.0 kg cart with a 2.0-m-long rope. The sides of the hill are inclined at 20°. He again keeps the rope parallel to the ground, and friction is negligible. What rope tension will allow the cart to spin with the same 14 rpm it had in the example?

69. ||| A small bead slides around a horizontal circle at height y inside the cone shown in FIGURE CP8.69. Find an expression for the bead's speed in terms of a, h, y, and g.

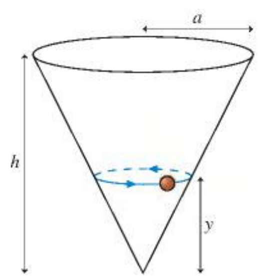


FIGURE CP8.69

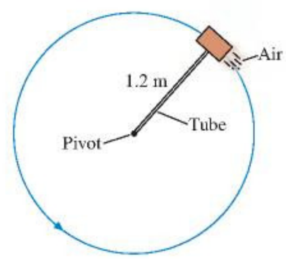


FIGURE CP8.70

70. ||| A 500 g steel block rotates on a steel table while attached to a 1.2-m-long hollow tube as shown in FIGURE CP8.70. Compressed air fed through the tube and ejected from a nozzle on the back of the block exerts a thrust force of 4.0 N perpendicular to the tube. The maximum tension the tube can withstand without breaking is 50 N. If the block starts from rest, how many revolutions does it make before the tube breaks?

71. ||| If a vertical cylinder of water (or any other liquid) rotates about its axis, as shown in FIGURE CP8.71, the surface forms a smooth curve. Assuming that the water rotates as a unit (i.e., all the water rotates with the same angular velocity), show that the shape of the surface is a parabola described by the equation $z = (\omega^2/2g)r^2$.

Hint: Each particle of water on the surface is subject to only two forces: gravity and the normal force due to the water underneath it. The normal force, as always, acts perpendicular to the surface.

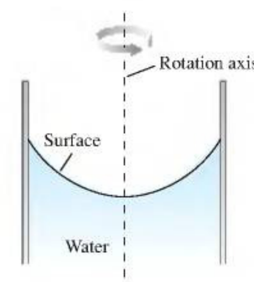


FIGURE CP8.71

Newton's Laws

KEY FINDINGS What are the overarching findings of Part I?

- **Kinematics** is the description of motion. Motion can be described
  - Visually
  - Graphically
  - Mathematically
- **Forces** cause objects to *change* their motion—that is, to accelerate.
- Objects **interact** by exerting equal but opposite forces on each other.

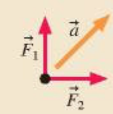
LAWS What laws of physics govern motion?

Newton's first law	An object will remain at rest or will continue to move with constant velocity if and only if $\vec{F}_{net} = \vec{0}$. The object is in **mechanical equilibrium**.	
Newton's second law	$\vec{F}_{net} = m\vec{a}$	A net force on an object causes the object to accelerate.
Newton's third law	$\vec{F}_{A\ on\ B} = -\vec{F}_{B\ on\ A}$	For every action, there is an equal but opposite reaction.

MODELS What are the most common models for applying the laws of physics to moving objects?

Constant force/Uniform acceleration

- Model the object as a particle.
  - Acceleration is in the direction of the net force and is constant.

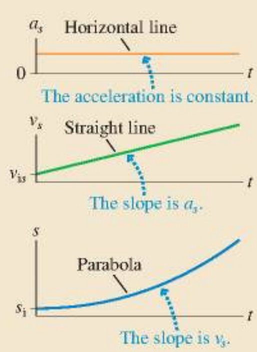


- Mathematically:
  - Newton's second law is

$$\vec{F}_{net} = \sum_i \vec{F}_i = m\vec{a}$$

  - Use *xy*-coordinates.
  - Constant-acceleration kinematics:

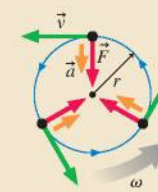
$$v_{fs} = v_{is} + a_s \Delta t$$
$$s_f = s_i + v_{is} \Delta t + \tfrac{1}{2} a_s (\Delta t)^2$$
$$v_{fs}^2 = v_{is}^2 + 2a_s \Delta s$$



- Special cases:
  - Uniform motion: $a_s = 0$. The displacement graph is a straight line with slope v_s.
  - Projectile motion: The only force is gravity. Horizontal motion is uniform; vertical motion has constant $a_y = -g$.

Central force/Uniform circular motion

- Model the object as a particle.
  - The force causes a constant centripetal acceleration. The particle moves around a circle at constant speed and with constant angular velocity.



- Mathematically: Newton's second law is
  - $\vec{F}_{net} = (mv^2/r$ or $m\omega^2 r$, toward the center)
  - Use *rtz*-coordinates.
  - Uniform-circular-motion kinematics:
    - The tangential velocity is $v_t = \omega r$.
    - The centripetal acceleration is v^2/r or $\omega^2 r$.
    - ω and v_t are positive for a ccw rotation, negative for a cw rotation.
- General case: Accelerated circular/rotational motion. Angular acceleration is contant.

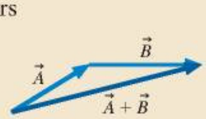
$$\omega_f = \omega_i + \alpha \Delta t$$
$$\theta_f = \theta_i + \omega_i \Delta t + \tfrac{1}{2}\alpha (\Delta t)^2$$
$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta\theta$$

These equations are analogous to constant-acceleration kinematics.

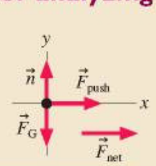
TOOLS What are the most important tools for analyzing the physics of motion?

- The particle model and motion diagrams

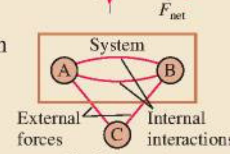
- Vectors



- Free-body diagrams



- Interaction diagrams



- Calculus and graphical analysis

$v_s = ds/dt =$ slope of the position graph

$a_s = dv_s/dt =$ slope of the velocity graph

$$v_{fs} = v_{is} + \int_{t_i}^{t_f} a_s\, dt = v_{is} + \text{ area under acceleration curve}$$

$$s_f = s_i + \int_{t_i}^{t_f} v_s\, dt = s_i + \text{ area under the velocity curve}$$